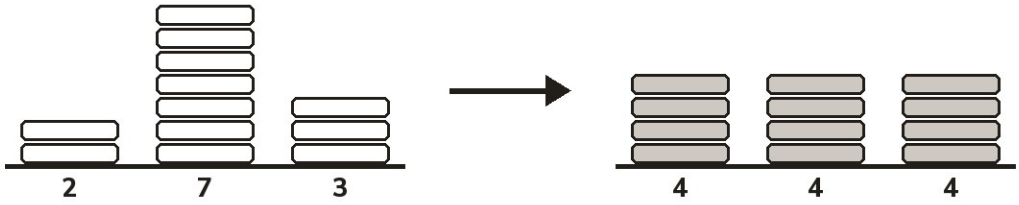
Does the average have to be one of the numbers? Helping your child see equal sharing

Parent page · ages 11–12 · goes with the video "Understand the concept"

A logical idea

Does your child say the average has to be one of the numbers? In everyday life, "average" describes something real, and in the first examples, the average often lands on one of the numbers (2, 3, 4: average 3). It isn't carelessness: they're figuring out what the word means.

The key idea



Before: 2, 7, 3
After: 4, 4, 4
Same total: 12 books

The average is what each one would get if we evened everything out, without adding or taking anything away. The calculation does this sharing: adding puts it all together (2 + 7 + 3 = 12); dividing shares it out equally (12 ÷ 3 = 4). Here, the average, 4, is none of the starting numbers.

Always true, and two traps

The average is never smaller than the smallest number, or larger than the largest: to even things out, you take from the tall stacks and give to the short ones. Traps: it isn't necessarily the middle number (1, 2, 9: average 4); it can be one of the numbers (6, 3, 1, 2: average 3); it doesn't tell you what each one had.

Questions instead of corrections

- "If we shared everything out equally, how many would each one get?"
- "Did the total change?"
- "Is your average between the smallest and the largest number?"

That last one helps catch a mistake, but doesn't prove the calculation. Then: "Tell me how you know."

The activities

Sixteen paperback books, in short stacks, on a sturdy table, out of reach of toddlers. Give your child only page 2: keep this parent page out of sight while they work.

A few days later, without warning (cut out)

Game 1: 7 points | **Game 2: 2 points** | **Game 3: 9 points**

What's your average per game? How do you know?

My average: _____

Read after your child has worked it out

- 2, 7, 3: 4 books per stack (12 books); 4 is none of the starting numbers.
- 2, 7, 3, and 4: still 4 per stack (16 books); this time, 4 is one of the numbers.
- 2, 7, 2, 3: 4 stacks of 3, with 2 books left over; average 3.5 (14 ÷ 4), between 2 and 7.
- Challenge: for example 3, 6, 6 or 1, 6, 8 or 4, 4, 7 (15 books in all).
- Cut-out: 7 + 2 + 9 = 18 and 18 ÷ 3 = 6, which is none of the scores.

A good sign: they talk about equal sharing or the total. If not, do the stacks again with other numbers: it often takes several tries.

Does the average have to be one of the numbers?



The average has to be one of the recorded numbers.

Do you agree with Idris?

1. I predict

Three stacks: 2, 7, and 3 books. If we make them equal: each stack will have ..... books.
Is it one of the starting numbers?

☐ yes ☐ no

Why? _____

2. I even them out

a. Make three stacks of books: 2, 7, and 3.
b. No books? Shade those stacks in pencil in the "Before" grid.
c. Start at the bottom. Use only the first three columns.
d. Copy your stacks into the "After" grid. Leave "Before" as is.
e. Move one book at a time, from one stack to another.
f. On paper, erase one shaded box, then shade another elsewhere.
g. Keep the total the same. Stop when the stacks are equal.

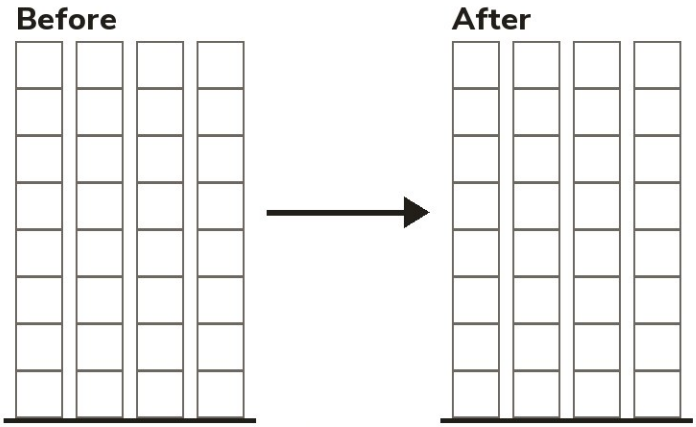
3. I check

Each stack has ..... books.
Total before: ..... Total after: .....
My prediction: ☐ was right ☐ needs changing

Changed your prediction? Great: you checked it yourself.

4. What if we add a stack of 4?

Start again with four stacks: 2, 7, 3, and 4 books. On the grids, erase everything. Shade these stacks in "Before", then copy them into "After". Even them out only in "After", using all four columns. Average: ..... Is it one of the starting numbers?



One box = one book. One column = one stack.

5. What if books are left over?

Make four stacks: 2, 7, 2, and 3 books. Predict the average: ..... Even them out without cutting any book. Books per stack: ..... Books left over: ..... In the box, draw 14 identical cookies. Share them equally among four groups. If needed, split drawn cookies into equal halves. Share those halves too. Each group gets ..... cookies. Average: .....

My 14 cookies

6. I explain

Can the average exceed the largest starting number? Explain why to an adult.

7. Family challenge

Find three stacks with an average of 5. None can have 5 books.
My stacks: ....., ..., ...